2014 Summer Review for Students Entering Geometry

1. Solving Linear Equations
2. Evaluating Expressions
3. Order of Operations
3. Operations with Rational Numbers
4. Laws of Exponents
5. Scientific Notation
6. Writing the Equation of a Line
7. Multiplying Polynomials
8. Solving Linear Systems of Equations
9. Interpreting Associations
10. Factoring Polynomials
11. Simplifying Radicals

(Suggested Timeline on Reverse Side)

2014 Summer Review for Students Entering Geometry

June 2014

Sun	Mon	Tues	Wed	Thur	Fri	Sat
	23 Complete Problems 1-5	24	25	26	27	28
29	30 Complete Problems 6-8					

July 2014

Sun	Mon	Tues	Wed	Thur	Fri	Sat
		1	2	3	4	5
6	7 Complete Problems 9-12	8	9	10	11	12
13	14 Complete Problems 13-15	15	16	17	18	19
20	21 Complete Problems 16-19	22	23	24	25	26
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August 2014

Sun	Mon	Tues	Wed	Thur	Fri	Sat
					1	2
3	4 Complete Problems 23-25	5	6	7	8	9
10	11 Complete Problems 26-30	12	13	14	15	16
17	18 Complete Problems 31-34	19	20	21	22	23
24	25 Complete Problems 35-40	26	27	28	29	30

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Order of Operations & Evaluating Expressions

The acronym PEMDAS may help: (Please Excuse My Dear Aunt Sally)

- First evaluate expressions in parentheses
- Evaluate each exponential (for example, $5^2 = 5 \cdot 5 = 25$).
- Multiply and divide each term from left to right.
- Combine like terms by adding and subtracting from left to right.

Numbers above or below a "fraction bar" are considered grouped. A good way to remember is to circle the terms like in the following example. Remember that terms are separated by + and – signs.

Example 1: Simplify $12 \div 2^2 - 4 + 3(1+2)^3$

Simplify within the circled terms: Be sure to perform the exponent operations before dividing.

$12 \div 2^2 = 12 \div 2 \cdot 2 = 3$

Then perform the exponent operation: $3^3 = 3 \cdot 3 \cdot 3 = 27$
Next, multiply and divide left to right: $3(27) = 81$
Finally, add and subtract left to right: $3 - 4 = -1$

$$12 \div 2^2 - 4 + 3(1+2)^3$$
$$3 - 4 + 3(3)^3$$
$$3 - 4 + 3(27)$$
$$3 - 4 + 81$$
$$-1 + 81$$
$$80$$

Example 2: Evaluate $5\left(\frac{x+2y}{x-y}\right)$ for $x = -3$, $y = 2$

$$5\left(\frac{-3+2\cdot 2}{-3-2}\right)$$
$$5\left(\frac{-3+4}{-3-2}\right)$$
$$5\left(\frac{1}{-5}\right) = -1$$

Operations with Rational Numbers

Use the same processes with rational numbers (positive and negative fractions) as are done with integers (positive and negative whole numbers).

Example 1: Compute $\frac{1}{3} + \left(-\frac{9}{20}\right)$

Solution: When adding a positive number with a negative number, subtract the values and the number further from zero determines the sign.

$$\frac{1}{3} + -\frac{9}{20} = \frac{1}{3}\cdot\frac{20}{20} + -\frac{9}{20}\cdot\frac{3}{3} = \frac{20}{60} + -\frac{27}{60} = -\frac{7}{60}$$

Example 2: Compute $-1\frac{1}{4} - \left(-3\frac{9}{10}\right)$

Solution: Change any subtraction problem to "addition of the opposite" and then follow the addition process.

$$-1\frac{1}{4} - \left(-3\frac{9}{10}\right) \Rightarrow -1\frac{1}{4} + 3\frac{9}{10} = -1\frac{5}{20} + 3\frac{18}{20} = 2\frac{13}{20}$$

Example 3: Compute $-1\frac{1}{4} \div 7\frac{1}{2}$

Solution: With multiplication or division, if the signs are the same, then the answer is positive. If the signs are different, then the answer is negative.

$$-1\frac{1}{4} \div 7\frac{1}{2} = -\frac{5}{4} \div \frac{15}{2} = -\frac{5}{4}\cdot\frac{2}{15} = -\frac{\cancel{5}\cdot\cancel{2}}{\cancel{2}\cdot 2\cdot 3\cdot\cancel{5}} = -\frac{1}{6}$$

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Laws of Exponents & Scientific Notation

The laws of exponents summarize several rules for simplifying expressions that have exponents. The rules are true for any base if $x \neq 0$.

$x^a \cdot x^b = x^{(a+b)}$ $\qquad$ $(x^a)^b = x^{ab}$ $\qquad$ $\frac{x^a}{x^b} = x^{(a-b)}$

$x^0 = 1$ $\qquad$ $x^{-a} = \frac{1}{x^a}$

Scientific notation is a way of writing a number as a product of two factors separated by a multiplication sign. The first factor must be less than 10 and greater than or equal to 1. The second factor has a base of 10 and an integer exponent.

Example 1: Simplify $4^2 \cdot 4^{-4}$

Solution: In a multiplication problem, if the bases are the same, add the exponents and keep the base. If the answer ends with a negative exponent, take the reciprocal and change the exponent to positive.

$4^2 \cdot 4^{-4} = 4^{-2} = \frac{1}{4^2} =$

Example 2: Simplify $\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y}$

Solution: Separate the fraction into two fractions with bases x and y. With an exponent on an exponent, multiply the exponents. Next, to divide expressions with exponents and the same base, subtract the exponents.

$$\frac{(x^2)^3 \cdot y^4}{x^{-2} \cdot y} = \frac{(x^2)^3}{x^{-2}} \cdot \frac{y^4}{y} = \frac{x^6}{x^{-2}} \cdot \frac{y^4}{y^1} = x^8 \cdot y^3 = x^8 y^3$$

Example 3: Multiply and give the answer in scientific notation. $(8 \times 10^4) \cdot (4.5 \times 10^{-2})$

Solution: Separate the number parts and the exponent parts. Multiply the number parts normally and the exponent part by adding the exponents. If this answer is not in scientific notation, change it appropriately.

$(8 \times 10^4) \cdot (4.5 \times 10^{-2}) = 8 \times 4.5) \cdot (10^4 \times 10^{-2}) = 36 \times 10^2 = (3.6 \times 10^1) \times 10^2 = 3.6 \times 10^3$

Multiplying Polynomials

Polynomials can be multiplied (changed from the area written as a product to the area written as a sum) by using the Distributive Property or generic rectangles.

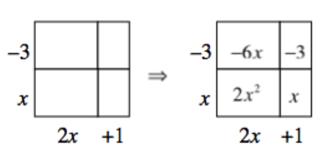
Example 1: Multiply $-5x(-2x + y)$

Solution: Using the Distributive Property

$-5x(-2x + y) = -5x \cdot -2x + -5x \cdot y = 10x^2 - 5xy$
area as a product $\qquad$ area as a sum

Example 2: Multiply $(x - 3)(2x + 1)$

Solution: Although the Distributive Property may be used, for this problem and other more complicated ones, it is beneficial to use a generic rectangle to find all the parts.



$\Rightarrow (x - 3)(2x + 1) = 2x^2 - 5x - 3$
area as a product $\qquad$ area as a sum

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Solving Equations

Equations in one variable may be solved in a variety of ways. Commonly, the first step is to simplify by combining like terms. Next isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable. Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 3 = x + 3$) it means that *all real numbers* are solutions.

Example 1: Solve $4x + 4x - 3 = 6x + 9$

Solution:

$4x+4x-3=6x+9$	problem
$8x-3=6x+9$	simplify
$2x=12$	add 3, subtract $6x$ on each side
$x=6$	divide

Check:
$$4(6)+4(6)-3=6(6)+9$$
$$24+24-3=36+9$$
$$48-3=45$$
$$45=45$$

Example 2: Solve for y: $\frac{y}{2}+\frac{y}{3}-3=y$

Solution:

$\frac{y}{2}+\frac{y}{3}-3=y$	problem
$6(\frac{y}{2})+6(\frac{y}{3})+6(-3)=6(y)$	multiply by the common denominator
$3y+2y-18=6y$	simplify
$-18=y$	subtract $5y$ from each side

Writing the Equation of a Line

Except for a vertical line, any line may be written in the form $y = mx + b$ where *"b"* represents the y-intercept of the line and *"m"* represents the slope. (Vertical lines are always of the form $x = k$.) The slope is a ratio indicating the steepness and direction of the line. The slope is calculated by . $m=\frac{\text{vertical change}}{\text{horizontal change}}=\frac{\Delta y}{\Delta x}$

Example 1: Write the equation of the line with slope $-\frac{1}{2}$ and passing through the point (6, 3).

Solution: Write the general equation of a line. $y = mx + b$

Substitute the values we know for m, x, and y. $3 = -\frac{1}{2}(6) + b$

Solve for b. $3 = -3 + b$

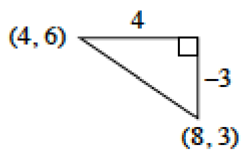
$6 = b$

Write the complete equation. $y = -\frac{1}{2}x + 6$

Example 2: Write the equation of the line passing through the points (8, 3) and (4, 6).

Solution: Draw a generic slope triangle

Calculate the slope using the given two points.



$\frac{\Delta y}{\Delta x}=\frac{-3}{4}$, $m=-\frac{3}{4}$

Write the general equation of a line. $y = mx + b$

Substitute m and one of the points for x and y, in this case (8,3). $3 = -\frac{3}{4}(8) + b$

Solve for b. $3 = -6 + b$

$9 = b$

Write the complete equation. $y = -\frac{3}{4}x + 9$

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Interpreting Associations (Scatterplots)

There are two reasons for modeling scattered data with a best-fit line. One is so that the trend in the data can easily be described to others without giving them the entire list of the data. The other is so that predictions can be made about points for which we do not have actual data.

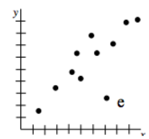
When describing an association between two variables, the form, direction, strength, and outliers should be described.

The **form** (shape) can be linear, or curved, and it can contain clusters or have gaps. Residual plots are an important part of determining the appropriateness of the form of the best-fit model. However, residual plots are not included in this checkpoint.

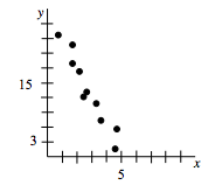
The **direction** is positive if one variable increases as the other variable increases, and negative if one variable decreases as the other increases. If there is no apparent pattern in the scatterplot, then the variables have no association. When describing a linear association, you can use the slope, and its numerical interpretation in context, to describe the direction of the association.

The **strength** is a description of how much the data is scattered away from the line or curve of best fit. An association is "strong" if there is little scatter about the model of best fit, and "weak" if there is a lot of scatter. When describing a linear association, the correlation coefficient, *r*, or *R*-squared can be used to numerically describe and interpret the strength.

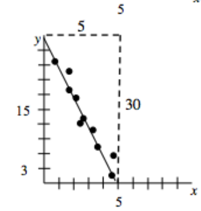
Outliers are data points that are far removed from the rest of the data.
A consistent best-fit line for data can be found by determining the line that makes the residuals, and hence the square of the residuals, as small as possible. We call this line the **least squares regression line** and abbreviate it LSRL. Your calculator can find the LSRL quickly.



Example 1: Describe the association in the scatterplot at right.
Solution: Looking from left to right, except for point (e), the points are fairly linear and increasing. This is a moderate, positive linear association. The point (e) is an outlier



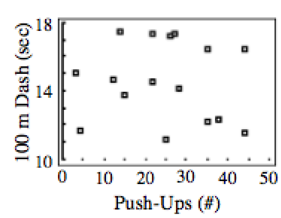
Example 2: For the scatterplot, draw a line of best fit and determine the equation of the line.
Solution: Use a ruler or straightedge to draw the line that approximates the trend of the points. If it is not a perfect line, approximately the same number of points will be above and below the line of best fit.



To determine the equation of the line, draw in a slope triangle and determine the ratio of the vertical side to the horizontal side. In this case it is $\frac{-30}{5} = -6$. Estimate the *y*-intercept by looking at where the line intersects the *y*-axis. In this case, it is approximately 30. The equation of any non-vertical line may be written in the form $y = mx + b$ where m is the slope and b is the *y*-intercept.

Example 3: Coach Romero is going to hold tryouts for the football team. Since timing students in the 100 m dash is time-consuming and inconvenient, he wondered if he could predict 100 m dash times from the number of push-ups a student can do. He went to the records from previous physical fitness exams and randomly chose a sample of students.

Push-Ups	100 m Dash (sec)	Push-Ups	100 m Dash (sec)
4	11.7	25	11.1
27	17.4	3	15.0
44	16.4	26	17.2
22	17.3	44	11.6
38	12.3	14	17.5
12	14.7	22	14.5
15	13.7	28	14.1
35	16.5	35	12.2
	checksums:	*394*	*233.2*



Use the data Coach Romero collected to describe the association between 100 m dash times and number of push-ups. Include an interpretation of the slope and *R*-squared. Then calculate the residual for the student that did 15 push-ups.

Solution: A statistical analysis always begins with a visual display of the data, in this case, a scatterplot. Start by entering the data into your calculator. Verify that you entered it correctly by comparing the sum of the values of each variable to the *checksum* value in the table above. Create reasonable axes for the scatterplot (often with a "windows" function).

The scatterplot shows no apparent pattern. In fact, the data seems randomly scattered. There is no association between time for the 100 m dash and the number of push-ups. This means that Coach Romero cannot reasonably use push-ups to predict 100 m dash times. It is not appropriate to create a best-fit line.

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Solving Linear Systems of Equations

When two equations are both in $y = mx + b$ form it is convenient to use the Equal Values Method to solve for the point of intersection. Set the two equations equal to each other to create an equation in one variable and solve for x. Then use the x-value in either equation to solve for y.

If one of the equations has a variable by itself on one side of the equation, then that expression can replace the variable in the second equation. This again creates an equation with only one variable. This is called the Substitution Method. See Example 1 below.

If both equations are in standard form (that is $ax + by = c$), then adding or subtracting the equations may eliminate one of the variables. Sometimes it is necessary to multiply before adding or subtracting so that the coefficients are the same or opposite. This is called the Elimination Method. See Example 2 below.

Sometimes the equations are not convenient for substitution or elimination. In that case one of both of the equations will need to be rearranged into a form suitable for the previously mentioned methods.

Example 1: Solve the following system. $4x + y = 8$
$x = 5 - y$

Solution: Since x is alone in the second equation, substitute $5 - y$ in the first equation, then solve as usual.

$$\begin{aligned} 4(5-y)+y &= 8 \\ 20-4y+y &= 8 \\ 20-3y &= 8 \\ -3y &= -12 \\ y &= 4 \\ x &= 5-4 \\ x &= 1 \end{aligned}$$

Then substitute $y = 4$ into either original equation to find x. Using the second equation $x = 1$ so the solution is (1, 4)

$$\begin{aligned} x &= 5 - 4 \\ x &= 1 \end{aligned}$$

Example 2: Solve the following system. $-2x + y = -7$
$3x - 4y = 8$

Solution: If we add or subtract the two equations no variable is eliminated. Notice, however, that if everything in the top equation is multiplied by 4, then when the two equations are added together, the y-terms are eliminated.

$$\begin{aligned} -2x+y&=-7 \\ 3x-4y&=8 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 4(-2x+y&=-7) \\ 3x-4y&=8 \end{aligned} \quad \Rightarrow \quad \begin{aligned} -8x+4y&=-28 \\ \underline{3x-4y}&\underline{=8} \\ -5x+0&=-20 \\ x&=4 \end{aligned}$$

Substituting $x = 4$ into the first equation $-2(4) + y = -7 \Rightarrow -8 + y = -7 \Rightarrow y = 1$.
The solution is (4,1).

Factoring Polynomials

Factoring polynomials requires changing a sum into a product. It is the reverse of multiplying polynomials and using a generic rectangle is helpful.

Example 1: Factor $x^2 + 7x + 12$

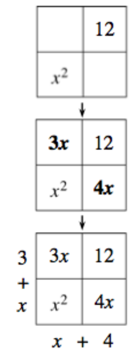
Solution: Sketch a generic rectangle with 4 sections.

Write the x^2 and the 12 along one diagonal.

Find two terms whose product is $12 \cdot x^2 = 12x^2$ and whose sum is $7x$. That is, $3x$ and $4x$. This is the same as a Diamond Problem from Chapter 1.

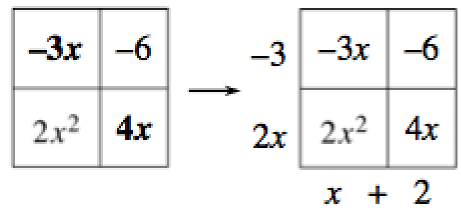
Write these terms as the other diagonal.
Find the base and height of the rectangle by using the partial areas.

Write the complete equation. $x^2 + 7x + 12 = (x + 3)(x + 4)$



Example 2: Factor $2x^2 + x - 6$.

Solution: Sketch a generic rectangle with 4 sections.
Write $2x^2$ and -6 along one diagonal.
Find two terms whose product is $-12x^2$ and whose sum is $1x$.
That is, $4x$ and $-3x$.
Write these terms as the other diagonal.
Find the base and height of the rectangle.
Write the complete equation. $2x^2 + x - 6 = (2x - 3)(x + 2)$



Simplifying Radicals

Before calculators were universally available, people who wanted to use approximate decimal values for numbers like $\sqrt{45}$ had a few options:

1. Carry around copies of long square-root tables.
2. Use Guess and Check repeatedly to get desired accuracy.
3. "Simplify" the square roots. A square root is **simplified** when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Simplifying square roots was by far the fastest method. People factored the number as the product of integers hoping to find at least one perfect square number. They memorized approximations of the square roots of the integers from one to ten. Then they could figure out the decimal value by multiplying these memorized facts with the roots of the square numbers. Here are some examples of this method.

Example 1: Simplify $\sqrt{45}$.

First rewrite $\sqrt{45}$ in an equivalent factored form so that one of the factors is a perfect square.

Simplify the square root of the perfect square. Verify with your calculator that both $3\sqrt{5}$ and $\sqrt{45}$ ≈ 6.71.

Example 1

$\sqrt{45} = \sqrt{9 \cdot 5}$
$= \sqrt{9} \cdot \sqrt{5}$
$= 3\sqrt{5}$

Examine **Example 2** and **Example 3** at right. Note that in Example 3, $\sqrt{72}$ was rewritten as $\sqrt{36} \cdot \sqrt{2}$, rather than as $\sqrt{9} \cdot \sqrt{8}$ or $\sqrt{4} \cdot \sqrt{18}$, because 36 is the largest perfect square factor of 72. However, since

$$\sqrt{4} \cdot \sqrt{18} = 2\sqrt{9 \cdot 2} = 2\sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \quad \text{and}$$
$$\sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2},$$

you can still get the same answer if you simplify it using different methods.

Example 2

$\sqrt{27}$
$= \sqrt{9} \cdot \sqrt{3}$
$= 3\sqrt{3}$

Example 3

$\sqrt{72}$
$= \sqrt{36} \cdot \sqrt{2}$
$= 6\sqrt{2}$

When you take the square root of an integer that is not a perfect square, the result is a decimal that never repeats itself and never ends. It is a number that cannot be written as a fraction using integers. This result is called an **irrational number**. The irrational numbers and the rational numbers together form the **real numbers**.

Generally, since it is now the age of technology, when a **decimal approximation** of an irrational square root is desired, a calculator is used. However for an exact answer, called **exact form** or **radical form**, the number must be written using the $\sqrt{\ }$ symbol.

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1. Factor $8x^2 + 28x + 12$ completely.

2. Factor each of the following expressions completely.

   a. $3x^2 + 9x + 6$ b. $100p^2 + 10p - 210$

3. Factor each of the special case quadratics below as completely as you can. Use a Diamond Problem and a generic rectangle for each one if possible.

   a. $16x^2 - 49$ b. $-19m + 56m^2 - 10$ c. $12y^2 - 3y$

4. Have you ever heard of the "Transylvania Effect"? This is the idea that during a full moon, more people are admitted to mental health facilities than at other times. The table below shows the average daily admittance rate (patients per day) for each month for one year.

Month	**Before Full Moon**	**During Full Moon**	**After Full Moon**
Aug.	6.4	5.0	5.8
Sept.	7.1	13.0	19.2
Oct.	6.5	14.0	7.9
Nov.	8.6	12.0	7.7
Dec.	8.1	6.0	11.0
Jan.	10.4	9.0	12.9
Feb.	11.5	13.0	13.5
Mar.	13.8	16.0	13.1
Apr.	15.4	25.0	15.8
May	15.7	13.0	13.3
June	11.7	14.0	12.8
July	15.8	20.0	14.5

Analyze this data. Is there any association between the full moon and the number of patients admitted?

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. Find the roots of each equation below. Use factoring, if possible.

a. $y = x^2 - x - 6$

b. $y = 3x^2 + 5x - 7$

c. $y = x^2 - 6x - 8$

d. $y = 2x^2 + x - 6$

6. Multiply: $(3x + 1)(x - 4)$.

7. Multiply: $(x - 6)(x^2 - 3x + 5)$.

8. Simplify each expression.

a. $-8\frac{2}{5} + (-3\frac{1}{8})$

b. $-6\frac{3}{7} - \frac{2}{9}$

c. $(-3\frac{3}{8})(5\frac{2}{9})$

d. $-1\frac{1}{9} \div (2\frac{6}{7})$

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9. Simplify each expression.

a. $12\frac{4}{7} + 4\frac{6}{14}$

b. $18\frac{16}{22} - 5\frac{8}{11}$

c. $(4\frac{13}{37})(5\frac{2}{7})$

d. $-92\frac{2}{5} \div (6\frac{3}{5})$

10. Is $x = 3$ the solution to the equation $8 + 3(x - 1) = 5x - 1$? How do you know? Justify completely.

11. Evaluate the expression $5x - \frac{6(3-x)}{x+7} + 3x^2$ for $x = -9$. Show your work.

12. Evaluate each expression if $x = 12$, $y = -2$, and $z = 0$.

a. $\frac{z}{xy}$

b. $3y^2 + 2x - 3z$

c. $x - y + z(x + y)$

d. $\left|x + |y|\right| + \sqrt{z} + xyz$

13. A student simplified each of the following using the order of operations. She did one of the four incorrectly. Which one? Once you have identified the incorrect simplification, help the student out by offering some feedback on what she might have done wrong and how to do the problem correctly.

a.
$9 \cdot 5 + 4(2 + (-3)) + 2 \cdot 9 =$
$= 9\ \ 5 + 4(-1) + 18$
$= 45 - 4 + 18$
$= 59$

b.
$2 \cdot 3 + 5 - (8 - 6) =$
$= 6 + 5 - 2$
$= 11 - 2$
$= 9$

c.
$(4 + 1)^2 + 10 - 3 + 2 =$
$= 5^2 + 10 - 3 + 2$
$= 25 + 10 - 3 + 2$
$= 35 - 5$
$= 30$

d.
$8 \cdot 2 + 3 \cdot (-2) - 5 \cdot 2 =$
$= 8 \cdot 2 + (-6) - 5 \cdot 2$
$= 16 - 6 - 10$
$= 0$

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14. A student simplified each of the following using the order of operations. Did he do any of them incorrectly? If so, which one? If you have identified an incorrect simplification, help the student out by offering some feedback on what he might have done wrong and how to do the problem correctly.

a. $2 \cdot (3 \cdot 4 + 8) + 6 \cdot 2 =$

$= 2 \cdot (12 + 8) + 12$
$= 2 \cdot 20 + 12$
$= 40 + 12$
$= 52$

b. $(4 \cdot 9) - (6 \cdot 3) \cdot 2 + 4 =$

$= 36 - 18 \cdot 2 + 4$
$= 36 - 36 + 4$
$= 4$

c. $7 + 5 \cdot (2 \cdot 3 + 6) \cdot 2 + 7 =$
$= 7 + 5 \cdot (6 + 6) \cdot 2 + 7$
$= 7 + 5 \cdot 12 \cdot 2 + 7$
$= 7 + 120 + 7$
$= 134$

d. $8 + 7 \cdot (6 + 18 \div 3) + 38 \div 2 =$
$= 8 + 7 \cdot (6 + 6) + 28 \div 2$
$= 8 + 7 \cdot 12 + 28 \div 2$
$= 8 + 84 + 14$
$= 106$

15. Put these numbers in order from smallest to largest.

$4 \quad 4^3 \quad 4^{-3} \quad 15 \quad 0 \quad -4 \quad \frac{1}{60} \quad \frac{1}{70} \quad -0.01$

16. Simplify:

a. $4x^3 \cdot x^3$

b. $\frac{14w^6}{7w^2}$

17. Simplify:

a. $m^3 \cdot m^4$

b. $\frac{h^8}{h^2}$

18. Simplify:

a. $\frac{\left(\frac{4}{3}ab^3\right)\left(\frac{6}{2}a^4\right)}{a^5}$

b. $\frac{8xy^8 \cdot 2}{48y^{10} \cdot \left(4y^2\right)}$

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19. A study team was discussing how to divide $6x^{10}$ by $12x^2$.

Andrea said, "The answer is $\frac{x^8}{2}$." Bernard said, "No, the answer is $2x^5$." Carmen disagreed and said, "No, the answer is $\frac{x^5}{2}$." Finally, Diego concluded, "You are all wrong. The answer is $2x^8$."

Are any of these students correct? How do you know? Explain well.

20. Rewrite the following numbers in scientific notation:

a. 0.0000000000000012

b. 7,600,000,000,000,000,000,000,000,000

21. Rewrite the numbers below either in scientific notation or in standard form.

a. 0.00000732

b. $5.49 \cdot 10^{11}$

c. $4.9 \cdot 10^{-4}$

d. 620,000,000

22. Find the equation of the line parallel to the line $-x + 2y = 12$ that goes through the point (10, 8). Show all work.

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23. Dominic is the star of the school basketball team. He makes many baskets during each game and could break the school record this season. Below is the data for the first five games of the season.

Game Number	Total Number of Baskets
1	6
2	8
3	15
4	22
5	29

a. Plot a graph with these data points.

b. Draw a trend line for this data using two carefully selected points that best represent the data.

c. Use the equation of your line to predict how many baskets Dominic will make by the end of the season if the season has 20 games.

24. Examine this tile pattern and answer the questions below.

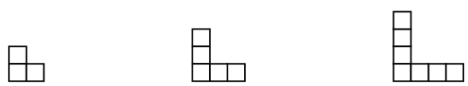

Figure 1 Figure 2 Figure 3

a. Write a rule that describes this pattern.

b. Which figure number has 41 squares? Show or explain how you figured it out.

25. Solve the following equations for the indicated variable. Show all of your work.

a. Solve for x: $-2(5x+10)=50$

b. Solve for y: $x+8y=24$

c. Solve for w: $4(4-t)=1+w-4$

d. Solve for x: $2x(x+3)=(2x-1)(x+4)$

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26. Hamal has a new part-time job bagging groceries after school. The equation $y = 10x + 50$ shows the relationship between his hours of work (x) and the amount of money in his bank account (y).

    a. How much money did he have in his bank account before he started working? How can you tell from the equation?

    b. How much is Hamal earning per hour? Justify your answer.

27. Moe and Larry each need to raise \$200 to go on a trip with their school band. Moe has \$35 in savings and earns \$20 per week delivering papers. Larry has \$60 in savings and earns \$10 per week doing yard work.

    a. Use at least two different methods to find the time (in weeks) when Moe and Larry will have the same amount of money in their savings accounts.

    b. How long will each boy have to work to have enough money for the trip?

28. Use tables, rules, and a graph to find and check the solution for the following problem.

    Edda has a poodle that weighs 7 pounds and gains 1 pound per year. Walden has a young sheltie that weighs 2 pounds and gains 1 pound every 6 months. When will the two dogs weigh the same amount?

29. Find the mistake in the solution below and explain how to correct it. Find the correct answer.

Algebra	Steps
$3x - 5 + 3x = x + 8 - x - 1$	original problem
$6x - 5 = 2x + 7$	combine like terms
$4x - 5 = 7$	subtract $2x$ from both sides
$4x = 12$	add 5 to both sides
$x = 3$	divide both sides by 4

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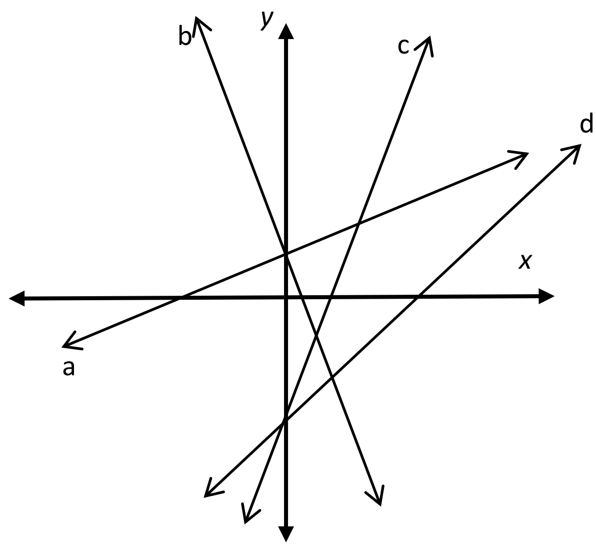
30. Mariela graphed all of the equations below but forgot which equation went with which graph. Help her match each equation with the appropriate graph. Discuss the answers with your group and write a few sentences explaining how you figured it out.

$y = x - 3$

$y = -2x + 1$

$y = \frac{1}{2}x + 1$

$y = 2x - 3$



31. Graph the following rules on one set of axes. Label each line with its equation, the *y*-intercept, and a growth triangle.

a. $y = 4x - 3$ b. $y = -2x + 5$ c. $y = -5x - 1$

32. Create a table, graph, and equation to match the tile pattern below. Use one of your representations to determine the number of tiles in the 100th figure.

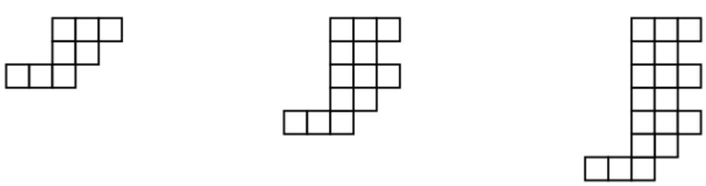


Figure 2 Figure 3 Figure 4

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33. Leonard earned the following chapter test scores during the first semester of freshman year: 78, 65, 92, 90, 82, 75. He would like to have a test average of 90%. If he has one test left to take, can he earn an average of 90%? Explain your reasoning.

34. Margie and Glenn are trying to solve the following system of equations.

$$5x - 4y = 23$$

$$7x + 8y = 5$$

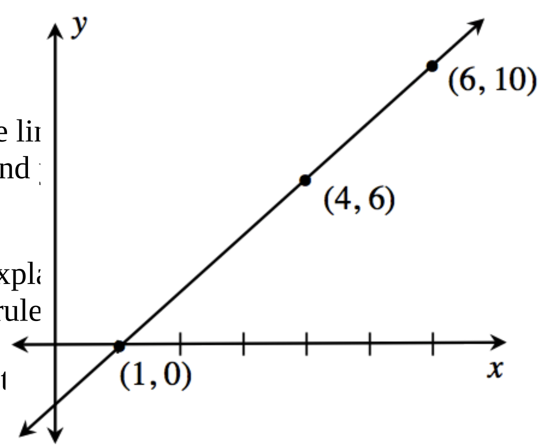
Glenn thinks that the elimination method will not work. Margie is not so sure about that. With your team, decide whether Glenn is correct and write a few sentences justifying your conclusion. If the elimination method will work, show how. If not, use another method to solve the system.

35. Three points are named on the line at the right.

a. Find three more points that lie on the li right. Show or explain how you found answer.

b. Find a rule for your line. Show or expl you and your group determined the rule

c. Verify your rule is correct using the t original points.



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36. The sum of two numbers is negative. The product of the two numbers is also negative. Explain everything you know about the two numbers. You may want include a diamond problem to explain your work.

37. Solve each of the following equations for x and check your work.

a. $1-(x+2)+3x=7$

b. $\frac{x+2}{1}=\frac{x+1}{2}$

38. Hypatia wants to use the elimination method to solve the system of equations below. She thinks that a variable will be eliminated if she multiplies the first equation by –1 and then adds the equations together. Is she correct? Explain your reasoning.

$$x + 2y = 8$$

$$y + x = 5$$

39. Claudia works at a sweet shop making chocolate-dipped bananas. Sometimes she drops the chocolate-dipped bananas before she can get them to the freezer. Claudia earns 20 cents for every frozen banana that she makes. She loses 5 cents for each frozen banana that she drops. Last Wednesday, Claudia dipped 48 frozen bananas, but not all of them made it to the freezer. She earned a total of $7.60. How many bananas were dropped and how many made it to the freezer? Define your variables and write a system of equations representing this situation. Solve the system by using either graphing, substitution, or elimination.

40. Simplify:

a. $\sqrt{20}$

d. $2\sqrt{3} + 3\sqrt{3}$

b. $3\sqrt{40}$

e. $3\sqrt{8} + 5\sqrt{2}$

c. $7\sqrt{12}$

f. $\sqrt{18} - 2\sqrt{27} + 3\sqrt{3} - 6\sqrt{8}$

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ANSWERS

1.) $4(2x + 1)(x + 3)$
2.) a: $3(x + 1)(x + 2)$, b: $10(5p – 7)(2p + 3)$
3.) a: $(4x + 7)(4x – 7)$, b: $(8m – 5)(7m + 2)$, c: $(3y)(4y – 1)$
4.) Answers will vary. There does appear to be an association.
5.) a: $x = 3$ and $–2$, b: $x = 0.9$ and $–2.6$, c: $x = 7.1$ and $–1.1$, d: $x = \frac{3}{2}$ and $–2$
6.) $3x^2 – 11x – 4$
7.) $x^3 – 9x^2 + 23x – 30$
8.) a: $-\frac{7}{18}$, b: $-\frac{461}{40}$, c: $-\frac{671}{63}$, d: $-\frac{141}{8}$
9.) a: 17, b: 13, c: 23, d: –14
10.) Yes, it is and students can justify by substituting 3 in for x and showing that each side is equal to the same amount.
11.) 234
12.) a: 0, b: 36, c: 14, d: 14
13.) Part (c) is incorrect because the student did not do the addition and subtraction left to right. They should have gotten 34.
14.) all are correct
15.) –4, –0.01, 0, 1/70, 4^{-3}, 1/60, 4, 15, 4^3
16.) a: $4x^6$, b: $2w^4$
17.) a: m^7, b: h^6
18.) a: $4b^3$, b: $\frac{x}{12y^4}$
19.) Andrea is correct.
20.) a: 1.2×10^{-15}, b: 7.6×10^{27}
21.) a: $7.32 \cdot 10^{-6}$; b: 549,000,000,000,000; c: 0.00048; d: $6.2 \cdot 10^3$
22.) $y = \frac{1}{2}x + 3$
23.) b: Answers will vary, but using data from games 2 and 4: $y = 7x$; c: about 140 baskets
24.) a: $y = 2x + 1$, b: figure 20
25.) a: $x = -7$, b: $y = -\frac{1}{8}x + 3$, c: $w = 19 - 4t$, d: $x = 4$
26.) a: \$50, with 0 hours of work Hamal has \$50 in his bank account; b: \$10 per hour is the growth rate
27.) a: After 2.5 weeks, both boys will have \$85, b: Moe will have to work 9 weeks, while Larry will have to work 14 weeks.
28.) The dogs will both weigh 12 pounds after 5 years.
29.) Like terms were not combined correctly in the second step ($x – x = 0$). The right side of the equation should have the 7 without the $2x$; $x = 2$.
30.) Line (a) is $y = \frac{1}{2}x + 1$. Line (b) is $y = -2x + 1$. Line (c) is $y = 2x - 3$. Line (d) is $y = x - 3$.
31.) Check for appropriate graphs with labels, y-intercepts, and triangles.
32.) $y = 5x – 2$, the 100th figure will have 498 tiles
33.) No, even if Leonard earned 100% on the last test, he'd still only have an overall test grade of 83%.
34.) It will work if you multiply every term in the first equation by 2, yielding $10x – 8y = 46$. The final answer is (3, –2).
35.) Answers vary. Points should follow the rule $y = 2x – 2$.
36.) Since the product of the two numbers is negative, one number is positive and the other is negative. Since the sum is also negative, the absolute value of the negative number is larger than the positive number.
37.) a: $x = 4$, b: $x = -3$
38.) Yes, Hypatia is correct. After multiplying, the first equation will have a $–1x$. Combining the equations eliminates the $–1x$ and the $1x$.
39.) Claudia made 48 bananas and dropped 8.
40.) a: $2\sqrt{5}$, b: $6\sqrt{10}$, c: $14\sqrt{3}$, d: $5\sqrt{3}$, e: $11\sqrt{2}$, f: $-9\sqrt{2} - 3\sqrt{3}$